A Low-Energy Direct Channel Regge-pole Approach to α-C^{12} Elastic Scattering. I

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Differential scattering cross-sections for the elastic scattering of α by C^{12} at laboratory bombarding energies from 11.0 to 16.0 MeV have been evaluated in the direct channel Regge-pole formalism, taking into account the contributions from a few nearby dominant excited levels of the compound nucleus O^{16} and incorporating the background effect. The relevant pole-parameters have also been predicted.

1. Introduction

The success of Regge-pole theory in high energy scattering processes has given rise to considerable interest in studying its application to nuclear scattering and reaction processes at low energy. It is to be noted here that of the methodologies advocated so far for nuclear scattering the most important ones are Optical Model [1], A.P.B.M. Model [2] and the Compound nucleus [3] analysis. However, the most remarkable triumph of the pole representation is that one needs not introduce phenomenological potential models.

Now, in the momentum plane (k-plane), in spite of the great success of the S-matrix theory in high energy physics, it has not been as successfull in nuclear scattering and reactions as the nuclear reaction theory based only on poles in the momentum plane is to some extent unrealistic because of some difficulties in the S-matrix representation in the momentum plane [4]. However, the elegance of the complex angular momentum (λ) approach is that it removes most of these difficulties and therefore provides a much more useful method for the analysis of nuclear scattering phenomena.

Now, the Regge simple pole formula [5] for low energy resonant scattering leads to a power law decrease of non-resonant scattering phases with angular momentum instead of an exponential decrease which is characteristic for forces with a finite effective radius. So, various modified pole-models have been suggested [6]. But one of the main drawbacks of these modified models is that they do not satisfy, explicitely, the elastic unitarity condition even if they give the proper asymptotic behaviour. The model which overcomes all these difficulties is the elegant and simple Modified Pole Model (MPM) of Grushin and Nikitin [7], who have taken into account the effect of the contributions from the left-half λ-plane by adding an integral to the simple Regge-pole formula and have successfully analyed the n-C^{12} elastic scattering phenomena with this model. The same model has already been successfully applied to charged particle scattering (C^{12}(d, d) C^{12}) at low energy without taking care of the Coulomb effect [8]. We have, therefore, taken recourse to this model in studying the resonant elastic scattering of a spinless projectile (α-particle) from a spinless target (C^{12}), taking into consideration the explicit "Coulomb-effect" in order to test the validity of the model to this type of nuclear scattering phenomena. It will be worth-mentioning here that, as informations about the crossed channel process are non-existent for this type of nuclear scattering processes, we Reggeize in the direct channel. The energy of the α-particles varies from 11.0 MeV (Lab.) to 16.0 MeV at 4 different randomly chosen energy values.

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2. Method of Calculations

The total amplitude for the elastic scattering of two spinless charged particles is given by

$$f_{\mathrm{T}}(E, Z) = f_{\mathrm{c}}(E, Z) + f_n(E, Z)\,, \tag{1}$$

where $f_{\mathrm{c}}(E, Z)$ is the "coulomb amplitude",

$$f_{\mathrm{c}}(E, Z) = -\frac{\eta}{2k}\operatorname{cosec}^2\theta/2 \cdot \exp\left(2i\eta \ln \operatorname{cosec}\theta/2\right), \tag{2}$$

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and $f_n(E, Z)$ is the so-called "nuclear amplitude", represented as

$$f_n(E, Z) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1)\, S_c(l, k) \cdot (\bar{S}(l, k) - 1)\, P_l(Z), \quad (3)$$

where $\bar{S}$ is the nuclear scattering-matrix, S_c the coulomb scattering-matrix, and η the coulomb charge-parameter.

Now, on Reggeizing the "nuclear amplitude" (3) we have

$$f_n(E, Z) = \frac{1}{2ik} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{(2\lambda+1) f(\lambda, E)\, S_c(\lambda, E)}{\sin \pi\lambda} \cdot P_\lambda(-Z)\, d\lambda + \frac{\pi}{k} \sum_{j=1}^{N} \frac{(2\lambda_j+1)\, R(\lambda_j, E)\, S_c(\lambda_j, E)}{\sin \pi\lambda_j} P_{\lambda_j}(-Z), \quad (4)$$

where λ_j is the jth Regge-pole given by $\lambda_j(E) = \alpha_j(E) + i\beta_j(E)$, $f(\lambda, E)$ the analytic continuation of the partial-wave amplitude with integral value of l to complex λ, and $R(\lambda, E)$ the residue of the partial-wave amplitude at λ, which is complex, in general.

Now, in the MPM [7] the back-ground integral is assumed to be negligibly small and the "nuclear-amplitude" takes the form

$$f_n(E, Z) = \frac{1}{k} \sum_{j=1}^{N} (2\lambda_j+1) \cdot R(\lambda_j, E)\, S_c(\lambda_j, E)\, P(\lambda_j, Z), \quad (5)$$

in which

$$P(\lambda, Z) = \frac{\pi}{\sin \pi\lambda} P_\lambda(-Z) + \int_0^{e^\xi} \frac{t^\lambda\, dt}{(t^2 - 2tZ + 1)^{1/2}},$$

when the leading poles are situated near some positive integral values of the angular momentum (corresponding to resonance states) close to the real axis in the complex angular momentum plane, (5) can be rewritten as

$$f_n(E, Z) = \frac{1}{k} \sum_{j=1}^{N} (2\lambda_j+1)\, R(\lambda_j, E)\, S_c(\lambda_j, E) \cdot \left(\frac{P_{l_j}(Z)}{\beta_j} + T_l(Z)\right), \quad (6)$$

where the $T_l(Z)$ are given by

$$T_0(Z) = \ln\left(\frac{c + \exp(\xi) - Z}{2}\right),$$

$$T_1(Z) = c + Z(1 + T_0),$$

and

$$l T_l(Z) = (2l-1) Z T_{l-1}(Z) - (l-1) T_{l-2}(Z) + \frac{1}{(2l-1)} \{P_l(Z) - P_{l-2}(Z)\} + c \exp(\xi(l-1)),$$

where

$$c = [\exp(2\xi) - 2Z \exp(\xi) + 1]^{1/2},$$

$$\operatorname{Cosh} \xi = 1 + t_0/2k^2 = 1 + \mu^2/2k^2.$$

Here "to" is the branch point in t-channel and μ is the mass of the lowest mass exchanged system.

The centre of mass differential cross-section is given by

$$\sigma(E, Z) = |f_T(E, Z)|^2. \quad (7)$$

Hence, the differential scattering cross-sections for the process can be evaluated with the help of (1), (2), (6) and (7).

The centre of mass differential scattering cross sections at incident laboratory energies of 11.00, 13.00, 14.00 and 16.00 MeV of alpha particles have been computed over the whole angular region by means of a least-squares fit and compared with the experimental results of Carter et al. [9]. At each energy value, the contributions from two or three nearby prominent resonances corresponding to the excited states of O^{16} [10] have been considered.

3. Results and Discussion

The angular distributions at laboratory energies of 11.00, 13.00, 14.00 and 16.00 MeV calculated from the pole representation (7) are shown along with the experimental ones in Figs. 1 to 4. The theoretical angular distributions, like the experimental ones, exhibit a distinct diffraction pattern. At all energies the angular distributions obtained from the pole representation reproduces all the experimental maxima and minima except at 11.00 MeV. At this energy, though fewer maxima and minima are obtained, nevertheless the overall agreement is quite satisfactory. At 14.00 MeV the third theoretical minimum is too deep while, except for the first minimum and maximum, the other theoretical maxima and minima are a little displaced towards lower angles.

However, over the entire energy range it has been found that 2 or 3 nearby leading poles are sufficient to account for the scattering phenomena at each

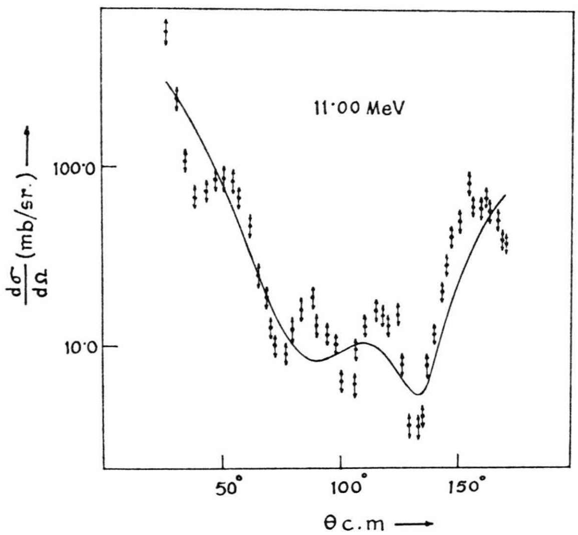

Fig. 1. Angular distribution (c.m.) at laboratory energy of 11.00 MeV (Lab.). Points indicate experimental values while the line represents the theoretical results.

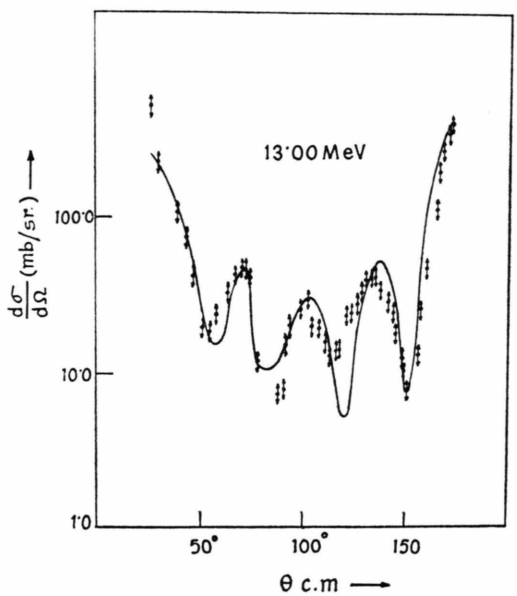

Fig. 2. Angular distribution (c.m.) at laboratory energy of 13.00 MeV (Lab.). Points indicate experimental values while the line represents the theoretical results.

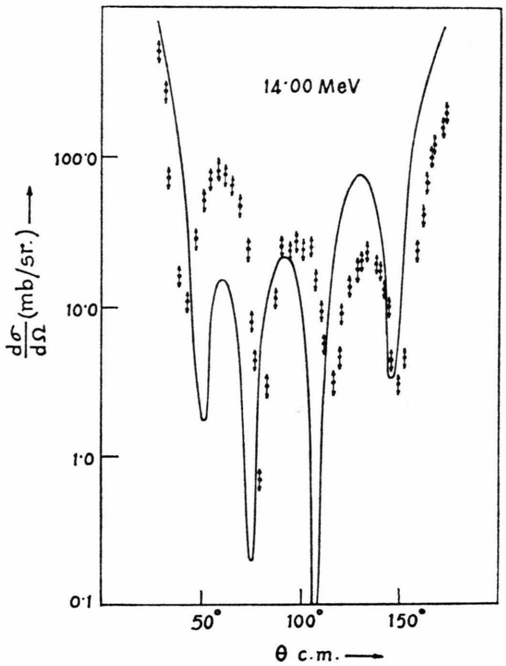

Fig. 3. Angular distribution (c.m.) at laboratory energy of 14.00 MeV (Lab.). Points indicate experimental values while the line represents the theoretical results.

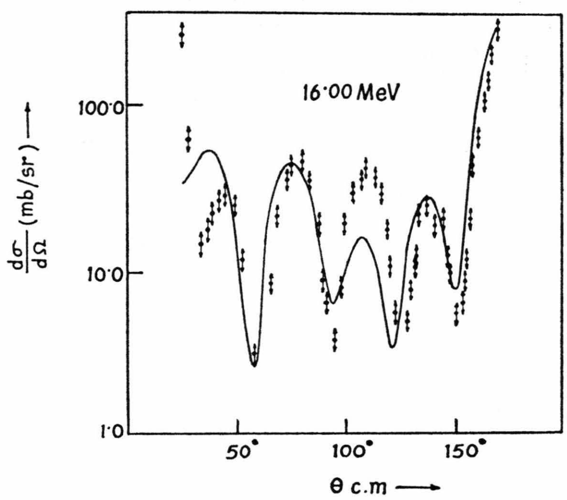

Fig. 4. Angular distribution (c.m.) at laboratory energy of 16.00 MeV (Lab.). Points indicate experimental values while the line represents the theoretical results.

energy value. The imaginary parts of the poles and the corresponding residues (both real and imaginary), evaluated by a least-squares fit to differential cross-section data, are tabulated along with their real parts and excitation energies in Table 1.

At 11.00 MeV the tentatively assigned levels (1^-) and (3^-) with excitation energy of 15.42 MeV have been confirmed by the present theoretical work. Again, at 16.00 MeV the existence of three resonance states

Table 1. Pole parameters at different energies.

E (Lab.) (MeV)	J^π	E_x (MeV)	β	Real (R)	Imaginary (R)
11.00	1^-	15.42	0.0193	0.4705	0.2784
	3^-	15.42	0.2272	1.1431	1.6968
	5^-	16.90	0.5289	9.4507	− 2.1345
13.00	2^+	16.94	0.5027	17.7280	− 4.1914
	4^+	16.80	0.8369	− 9.0626	3.5633
	0^+	17.70	0.0074	−82.8470	−12.6520
14.00	2^+	17.70	0.0012	2.4868	0.7216
	4^+	17.81	0.1576	428.960	48.2420
	2^+	19.12	0.0037	0.4734	0.3099
16.00	4^+	19.12	0.0450	− 0.4124	7.6517
	5^-	19.25	0.0021	− 0.6075	0.0319

$$2^+(E_x = 19.12 \text{ MeV}), \quad 4^+(E_x = 19.12 \text{ MeV})$$

and

$$5^-(E_x = 19.25 \text{ MeV})$$

of the compound nucleus O^{16}, which were tentatively assigned from experiment [10], are supported by this work. Hence the present work gives information about the excited states of O^{16}. However, we conclude that, if the effect of the background, i.e. the effect of the singularities of the partial-wave amplitude in the left-half λ-plane is taken into account, then two or three nearby dominant Regge-poles are sufficient to reproduce the essential features of the low energy nuclear charged-particle scattering of a spinless projectile by a spinless target. That is to say that we have been able to reproduce in a very simple and elegant way the low energy $C^{12}(\alpha, \alpha)\,C^{12}$ angular distribution data and have obtained information about the excited states of O^{16} along with the corresponding pole-parameters by means of a very simple modified pole representation, which satisfies all the physical features required by the partial-wave amplitude in the complex angular momentum plane. On the other-hand, the present analysis proves conclusively that the Modified Pole Model of Grushin and Nikitin is well applicable in the case of spinless charged-particle scattering processes.

All the required computational work has been performed on a Burroughs-6700 computer at the Regional Computer Centre, Calcutta.

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